BIRZEIT UNIVERSITY

Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Engineering Probability and Statistics ENEE 2307

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Midterm Exam

Date: Sunday 4/12/2016 Time: 75 minutes
Name: Student #:

Opening Remarks:

- This is a 75-minute exam. Calculators are allowed. Books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.

Problem 1 (20 pts)**:**

a. If a multiple-choice test consists of 5 questions, each with 4 possible answers of which only one is correct. Assume a student just randomly guesses (يتحزر) the correct answer to each questions. What is the probability that the student gets all of them wrong?
b. A pair of coins are tossed simultaneously and independently. Each coin has a probability 0.55 to be heads (H). What is the probability that the outcomes of the two coins are different?

Problem 2 (15 pts)

In an experiment to study the relationship of hypertension (الضغط) and smoking habits, the following data are collected:

	Nonsmokers (NS)	**Moderate Smokers (MS)**	**Heavy Smokers (HS)**
Hypertension **(H)**	15%	19%	16%
No-hypertension **(NH)**	25%	15%	10%

a. What is the probability that a randomly selected person is a Nonsmoker?
b. What is the probability that a randomly selected person is both a moderate smoker and experiences hypertension?
c. If a random person is selected and found to be a heavy smoker, what is the probability that the person is experiencing hypertension?

Problem 3 (16 pts)

The waiting time, in hours, between successive speeders (المتجاوزين للسرعة) spotted by a radar unit is a continuous random variable with cumulative distribution function

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-8x} & x \geq 0 \end{cases}$$

a. Find the probability of waiting less than 12 minutes between successive speeders?
b. What is the average waiting time, in hours, between successive speeders?

Problem 4 (16 pts)**:**

In testing a certain kind of truck tire, it is found that 25% of the trucks fail to complete the test run without a blowout.

a. Find the probability that out of 6 trucks tested, less that two have blowouts.
b. How many of the 6 tested trucks would you expect to have blowouts?

Problem 5 (15 pts)

Suppose that the proportion of colorblind people in a large population is 0.005. Use the normal approximation to calculate the probability that there will be at most 32 colorblind person in a randomly chosen group of 6000 people.

Problem 6 (18 pts)**:**

Let X be a random variable representing the time (in years) it takes to develop a software. Suppose that X has the following probability density function

$$f_X(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ 0 & otherwise \end{cases}$$

a. Find k so that this is a valid probability density function
b. Compute the probability that it takes more than 1 year to develop the software.
c. Find the probability that it will take more than 6 months to develop the software given that it already exceeded 3 months?

Good Luck

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Midterm Exam — Sunday, December 4th 2016

1. a. $P(T) = 1/4$, $P(F) = 3/4$

$P(FFFFF) = P(F)^5 = \left(\frac{3}{4}\right)^5 = 0.2373$

b. $P(H) = 0.55$, $P(T) = 0.45$

$S = \{HH, HT, TH, TT\}$;

$P(\text{Different}) = P(HT) + P(TH) = 2P(HT) = 2(0.55) * (0.45)$

$= 0.4995$

Problem 2.

	Nonsmokers (NS)	Moderate smokers (MS)	Heavy smokers HS	
Hypertension (H)	15%	19%	16%	
Non-hypertension (NH)	25%	15%	10%	

a. $P(\text{Nonsmoker}) = (0.15) + (0.25) = 0.4$

b. $P(MS \cap H) = 0.19$

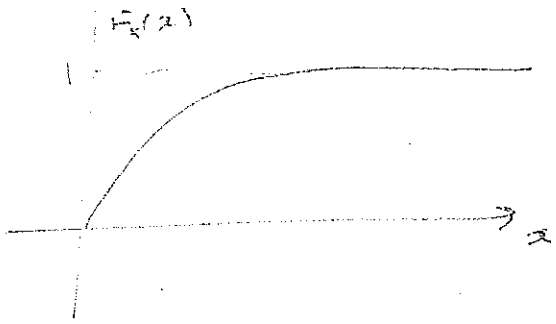
c. $P(H/HS) = \frac{P(H \cap HS)}{P(HS)}$

$= \frac{0.16}{(0.16) + 0.10} = \frac{0.16}{0.26} = \frac{16}{26}$

$= \frac{8}{13} = 0.615$

Problem 3

$$F_x(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-8x} & x \geq 0 \end{cases}$$



a. X: waiting time (in hours)

$$P\left(X \leq \frac{12}{60}\right) = P\left(X \leq \frac{1}{5}\right) \quad ; 12 \text{ minutes}$$

$$= F\left(\frac{1}{5}\right) = 1 - e^{-8/5} = 1 - e^{-1.6}$$

b. $$E(X) = \frac{1}{\lambda} = \frac{1}{8} \text{ (hr)} = \frac{60}{8} = 7.5 \text{ minutes}$$

b. $$f_x(x) = 8e^{-8x} \quad x \geq 0$$

$$E(X) = \int_0^\infty x\left[8e^{-8x}\right] dx = \frac{1}{8} \text{ hr} \Rightarrow 7.5 \text{ minutes.}$$

Problem 4

~~$P(X=1) = 0.25 \quad P(X=5) = 0.75$~~ $P = 0.25$

$1 - P = 0.75$

a. X: # of trucks that fail the test

$$P(X < 2) = P(X=0) + P(X=1)$$

$$= \binom{6}{0} P^{(0)}(1-P)^6 + \binom{6}{1}(P)^1(1-P)^{6-1}$$

$$= (0.75)^6 + 6(0.25)(0.75)^5$$

b. $$E(X) = nP$$

$$= (6)(0.25) = 1.5$$

Problem 5

$p = 0.005$

$n = 6000$

x: number of colorblind persons $\quad 6000 - x$

$$P(X \leq 32) = \sum_{0}^{32} \binom{6000}{x} p^x (1-p) \qquad \text{[exact]}$$

normal approximation

$$\mu_x = np = (6000)(0.005) = 30$$

$$\sigma_x^2 = np(1-p) = 6000 * 0.005 * (1 - 0.005) = 29.85$$

$$P(X \leq 32) = \Phi\left(\frac{32 - 30}{\sqrt{29.85}}\right) = \Phi(0.366)$$

$$= 0.64$$

Problem 6

$$f_x(x) = \begin{cases} k x^2 & 0 < x < 2 \\ 0 & \text{o.w} \end{cases}$$

x ; in years

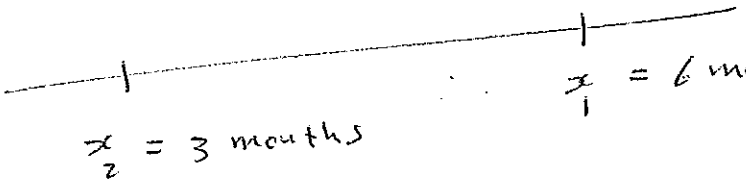
a. $\int_0^2 k x^2 dx = k \frac{x^3}{3}\Big|_0^2 = k\left(\frac{8}{3}\right) = 1 \Rightarrow k = \frac{3}{8}$

b. $P(X>1) = \int_1^2 k x^2 dx = k \frac{x^3}{3}\Big|_1^2 = \frac{3}{8} \cdot \frac{(8-1)}{3} = \frac{7}{8}$

c. X ; in years

$$P(X > x_1 / X > x_2) = \frac{P(X > x_1 \cap X > x_2)}{P(X > x_2)}$$

$$= \frac{P(X > x_1)}{P(X > x_2)}$$



$x_1 = \frac{6}{12} = 0.5$ year

$x_2 = \frac{3}{12} = 0.25$ year

$$= \frac{\int_{0.5^-}^{2} k x^2 dx}{\int_{0.25^-}^{2} k x^2 dx} = \frac{x^3/3\Big|_{0.5^-}^{2}}{x^3/3\Big|_{0.25^-}^{2}}$$

$$= \frac{(8 - (0.5^-)^3)}{(8 - (0.25^-)^3)}$$

$$= \frac{(8 - 0.125^-)}{(8 - 0.015625^-)} = \frac{7.875^-}{7.98443} =$$

$$= 0.9863$$